Exploiting Problem Structure in Combinatorial Landscapes: A Case Study on Pure Mathematics Application

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SUMMARY
- Use AI techniques to find narrow admissible tuples, a case of pure mathematics applications
  - Formulate the original problem into a combinatorial optimization problem
  - Exploit the local search structure for reduction in search space & elimination in search barriers
  - Realize search strategies for tackling problem structure & escaping from local minima
- Shed light on exploiting the local problem structure for efficient search in combinatorial landscapes as an application of AI to a new problem domain

RESULTS

- Results by Existing Methods: [Polymath, 2014a; 2014b]
  - Most methods are constructive and sieve methods
  - Empirically, the bound is $H(k) \leq k \log k + o(1)$
- Online database [Sutherland, 2015] for $k \leq 5000$
  - $k$ primes past $k$
    - 8424 18386 29792 39660 50840
    - Eratosthenes 8212 17766 20808 38596 49578
    - Schinzel 8336 18126 20802 38418 49056
    - Hensley-Richards 8285 17726 27006 38498 48634
    - Shifted Schinzel 8190 17716 27500 37782 48282
- Best known
  - 7802 16978 26606 36610 46806

RESULTS by RALS Algorithm: Different Parameters
- $T = 0$: The best results by the shifted greedy sieve
- $L, N_{12}, N'_{12}$: Local search with different levels, iterations, and contraction depths
- $\gamma$: The region selection with different randomizations

Future Work
- Deeper analysis to identify more local search properties
- Toward general optimization with advanced AI strategies